SYSTEMS OF LINEAR EQUATIONS

A **system of linear equations** (or linear system) is a collection of linear equations involving the same set of variables.

Solving a system of equations means finding the values of the variables that make all the equations true at the same time.

Example: $\begin{cases} 2x - 3y = 3 \\ 5x + 3y = 18 \end{cases}$ $\Rightarrow$ The solution to this system is $x = 3$ and $y = 1$.

Number of solutions of two-variable systems

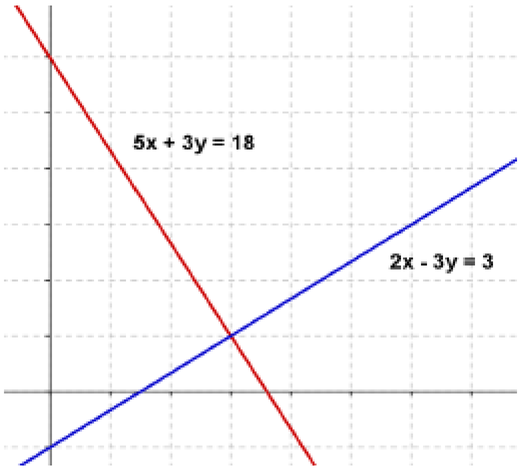
When you are solving systems, you are, graphically, finding intersection of lines. (Remember that the graph of a linear equation, $ax + by = c$, is a straight line, and its points are the solutions ot the equation).

For two-variable systems, there are three possible types of solutions:

<u>Case 1</u>

$$\begin{cases} 2x - 3y = 3 \\ 5x + 3y = 18 \end{cases}$$

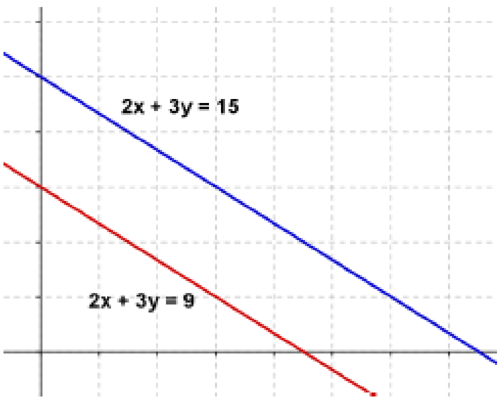
The two lines cross at exactly one point. This point is the **only solution** to the system. These systems are called **independent systems**.



<u>Case 2</u>

$$\begin{cases} 2x + 3y = 15 \\ 2x + 3y = 9 \end{cases}$$

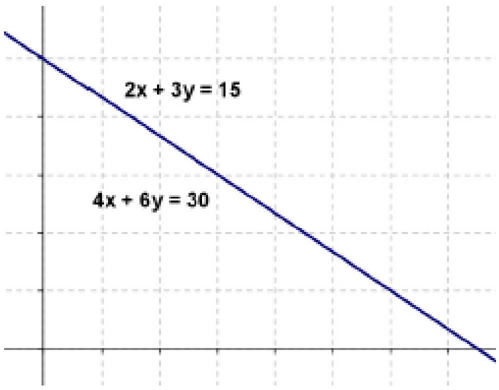
Since parallel lines never cross, the system has **no solution**. These systems are called **inconsistent systems**.



<u>Case 3</u>

$$\begin{cases} 2x+3y=15 \\ 4x+6y=30 \end{cases}$$

The two lines are the same line. **Any point of the line is solution** to the system. These systems are called **dependent systems**.



Methods to solve systems of linear equations

<u>The graphical method</u>

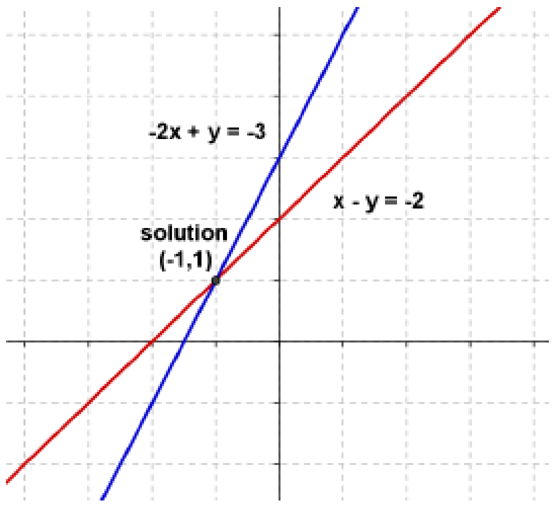
The graphical method consists of graphing every equation in the system and then using the graph to find the coordinates of the point(s) where the graphs intersect. The point of intersection is the solution.

Example: Use the graphical method to solve the following system of equations.

$$\begin{cases} x-y=-2 \\ -2x+y=3 \end{cases}$$

Graph both equations very precisely. If you don't graph neatly, your point of intersection will be way off.

The **solution** is $x=-1$, $y=1$. Substitute these values into both equations to check the solution.



Use the graphical method to solve the following $\begin{cases} 2x-3y=0 \\ x+3y=9 \end{cases}$

The substitution method

To solve a system of equations using substitution, first solve one of the equations for a variable, then substitute it into the other equation. Always substitute into the other equation and always use parentheses.

Example: Use the substitution method to solve the following system of equations.

$$\begin{cases} 3x+4y=8 \\ -2x+5y=3 \end{cases}$$

The solution is $x=\frac{28}{23}$, $y=\frac{25}{23}$.

The elimination method

Elimination consists of adding equations together to eliminate variables. Sometimes you have to multiply equations by a number before you add them. The goal is to end up with one equation that has just one variable. Then you can use back-substitution to solve for the other variable.

When using elimination, eliminate one variable at a time. It is also important to write down "instructions" that indicate how you are manipulating the equations going from step to step.

Example: Use the elimination method to solve the following system of equations.

$$\begin{cases} 5x+3y=-7 \\ 4x+5y=-3 \end{cases}$$

The solution is $x=-2$, $y=1$.

The equating method

Equating consist of isolating the same unknown in both equations and then equate both expressions. Solve the equation with one variable and use back-substitution ~~tos ove~~ for the other variable.

Example: Use the equating method to solve the following system $\begin{cases} y=x-1 \\ 2(x-1)+y=3y \end{cases}$

Infinite solutions.

Solve the following systems of equations.

a) $\begin{cases} \dfrac{x}{3} - \dfrac{y}{2} = 4 \\ \dfrac{x}{2} - \dfrac{y}{4} = 2 \end{cases}$

b) $\begin{cases} \dfrac{x+15}{8} + \dfrac{3(y+1)+y}{16} = 3 \\ \dfrac{7-x}{2} - \dfrac{1+y}{12} = 3 \end{cases}$

Solve the following systems of equations with three variables.

a) $\begin{cases} x - y = 0 \\ x - 2z = 6 \\ y + z = 3 \end{cases}$

b) $\begin{cases} x - z = 4 \\ 2x + y = 7 \\ x + y = 2z \end{cases}$

OTHER TYPES OF SYSTEMS OF EQUATIONS

Up to now, we have solved systems of linear equations. However, the substitution method and the elimination method allow us to solve other types of systems.

Examples:

a) $\begin{cases} y - x = 1 \\ x^2 + y^2 = 5 \end{cases}$ We will use the substitution method.

Solve for "y" in the first equation: $y = x + 1$

Substitute the previous expression of "y" into the second equation:

The solution to the system is: $\begin{cases} x_1 = 1, \quad y_1 = 2 \\ x_2 = -2, \quad y_2 = -1 \end{cases}$

b) $\begin{cases} x^2 + y^2 = 58 \\ x^2 - y^2 = 40 \end{cases}$ We will use the elimination method. add both equations

The solution to the system is: $\begin{cases} x_1 = 7, \quad y_1 = 3 \\ x_2 = 7, \quad y_2 = -3 \\ x_3 = -7, \quad y_3 = 3 \\ x_4 = -7, \quad y_4 = -3 \end{cases}$

c) $\begin{cases} \frac{1}{x} + \frac{1}{y} = 1 - \frac{1}{xy} \\ xy = 6 \end{cases}$ We start by simplifying the first equation.

Therefore, the equivalent system that we get is: $\begin{cases} y + x = xy - 1 \\ xy = 6 \end{cases}$

substitute the value of "xy" of the second equation into the first equation,

The solution to the system is: $\begin{cases} x_1 = 3, \quad y_1 = 2 \\ x_2 = 2, \quad y_2 = 3 \end{cases}$

Solve the following systems of equations.

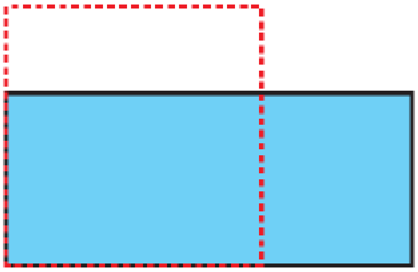
a) $\begin{cases} x^2 + xy + y^2 = 21 \\ x + y = 1 \end{cases}$

b) $\begin{cases} \dfrac{1}{x} + \dfrac{1}{y} = \dfrac{1}{20} \\ x + 2y = 3 \end{cases}$

c) $\begin{cases} 2\sqrt{x+1} = y + 1 \\ 2x - 3y = 1 \end{cases}$

The perimeter of a rectangle is 34 m and its area is 60 m^2. Find the dimensions of this rectangle.

Consider a rectangular plot. It its base decreased in 80 m and its height increased in 40 m, it would turn into a square. If its base decreased in 60 m and its height increased in 20 m, then its area would decrease in 400 m^2. Find the dimensions of the plot.



The side of a rhombus is 5 cm and its area is 24 cm^2. Find the dimensions of the diagonals.

The hypotenuse of a right triangle is 16 cm longer than the shortest side and 2 cm longer than the remaining side. Find the dimensions of this triangle.

Frank's Specially Coffees makes a house blend from two types of coffee beans, one selling for \$9.05 per pound, and the other selling for \$6.25 per pound. If he makes 15 pounds of house blend and sells them for \$7.37 per pound, how many pounds of each type of coffee does he need to make his house blend?

You are selling tickets for a high school basketball game. Student tickets cost $3 and general admission tickets cost $5. You sell 350 tickets and collect $1450. How many of each type of ticket did you sell?

An investor buys two paintings for $2650. In two years' time, he sells them for $3124 and makes a profit of 20% in the first painting and 15% in the second one. How much did the investor pay for each painting?

Three consecutive even integers are such that the square of the third is 100 more than the square of the second. Find the three integers.

A pilot flies 630 miles with a tail wind of 35 miles per hour. Against the wind, he flies only 455 miles in the same amount of time. Find the rate of the plane in still air.

Rudy must play 12 commercials during his 1 hour show. Each commercial is either 30 seconds or 60 seconds long. If the total commercial time during that hour is 10 minutes, how many commercial of each type does Rudy play?

A group of friends rent a van for 490 € to go on a trip. Since two more friends decide to go at the last moment, each of the others is refunded 28 €. How many friends go on that trip and how much does each one pay?